

$$\text{sharpness}(P) = \frac{H}{W} = \frac{V_3 - V_1 + V_2}{I_3 - I_1}$$

and

$$\text{magnitude}(P) = \int_{I_1}^{I_3} v(i) di - N_i(P),$$

$$\text{where } N_i(P) = \frac{(V_1 + V_2) \times (I_3 - I_1)}{2}.$$

Each peak can be represented by fuzzy languages. The membership functions for the employed fuzzy languages are given in Figure 3. Thus the histogram is recognized as the collection of the peaks whose attributes are represented by the fuzzy languages. Figure 4 shows our histogram models, Model A and Model B, able to concentrate various histograms. Table 1 shows the knowledge representation of the models. We can derive the granules and can find two thresholds by comparing an input histogram with our histogram models by the following steps.

Table 1 Fuzzy membership functions of the models.

Granule	Histogram Model A		Histogram Model B	
	<i>sharpness</i>	<i>magnitude</i>	<i>sharpness</i>	<i>magnitude</i>
G _g	rather big	rather big	rather big	rather big
G _w	big	big	big	big
G _c	not exist	not exist	rather small	rather small

- Step 1) Derive all possible patterns for evaluating which possibility, Model A or Model B, is higher for the input.
- Step 2) For the each peak, compute *sharpness* and *magnitude*.
- Step 3) Compute a minimum between *attribute* values, *sharpness* and *magnitude*, and the corresponding membership functions for each peak. We obtain the average values, A_s (for *sharpness*) and A_m (for *magnitude*). Similarity measure, A_s + A_m, is calculated. Pick up the pattern with the maximum similarity measure and employ the model that the pattern belongs to.

The interested histogram is thus classified into a model that has the maximum similarity measure. From the obtained peaks specifying the input histogram, we can obtain the required thresholds.

2.2 Experimental Result

We segment 50 human brain MR volumes by the obtained thresholds and calculate their volumes. The results are compared with the manually segmented ones by a medical doctor. The manually segmented results are obtained by tracing their outlines for every image slice of the MR volumes. The segmentation accuracy obtained by averaging the absolute values of error ratio was 2.3% for 50 MR volumes. The standard deviation was 0.22.

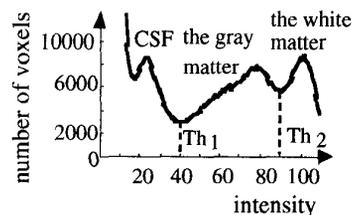


Figure 1 A histogram of brain MR image.

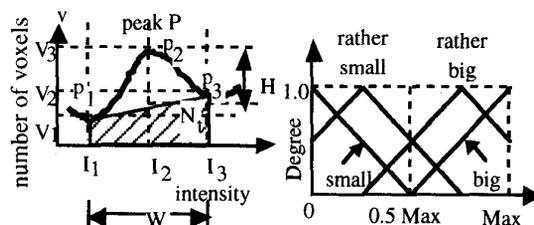


Figure 2 An Illustration. Figure 3 Membership functions

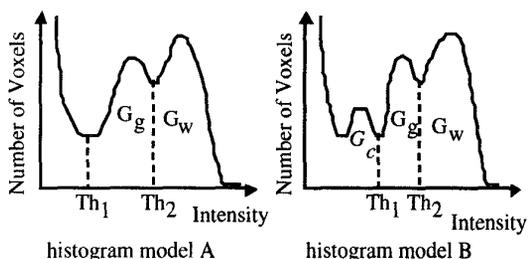


Figure 4 Two histogram models.

3. SEGMENTATION BY FUZZY INFERENCE

3.1 Fuzzy Inference on Image Segmentation

Image segmentation needs to help with human knowledge. We consider possible knowledge to segment the ROIs and we will represent the obtained information by fuzzy if-then rules. Consider to segment several ROIs in an image. We represent the entire knowledge residing within a collection of rules to segment the ROIs:

- Knowledge 1:
 If $x_{mn}=A_1$ then $x_{mn}=G_{11}$.
 Else if $x_{mn}=A_2$ then $x_{mn}=G_{12}$, ...
 Knowledge p: (p=2, 3, ...)
 If $x_{mn}=C_1$ then $x_{mn}=G_{p1}$.
 Else if $x_{mn}=C_2$ then $x_{mn}=G_{p2}$, ...

Note that the all rules are evaluated to achieve same goal: segmenting the ROIs. The membership values of G' are calculated by:

From Knowledge 1:

$$\mu_G^1(x_{mn}) = \max_{m \times n \in M \times N} \min(\mu_A^1(x_{mn}), \mu_R(x_{mn}, x_{mn})).$$

From Knowledge p (p=2, 3,...)

$$\mu_G^p(x_{mn}) = \max_{m \times n \in M \times N} \min(\mu_C^p(x_{mn}), \mu_R(x_{mn}, x_{mn})).$$

Next step is constructing the pixels to ROIs. We should effectively and selectively employ the inference results

$\mu_G^1(x_{mn}), \dots, \mu_G^p(x_{mn})$. The typical constructing techniques are known in labeling, region growing or clustering techniques.

3. 2 Application to MR and CT Images

3.2.1 Human Brain MR Images

Section 3.2.1 applies the image segmentation to MR brain images. Assume that the 124 two-dimensional MR images are arranged in order along the z-axis as shown in Figure 5. The ROIs in this subsection are: the left cerebral hemisphere(LCH for short), the right cerebral hemisphere(RCH), the cerebellum(CB) and the brain stem(BS). For a given 3D MR image we can obtain the whole brain region in Section 2. For the obtained brain data, assume the center of gravity of the brain region as the origin and its coordinate system is shown in Figure 5.

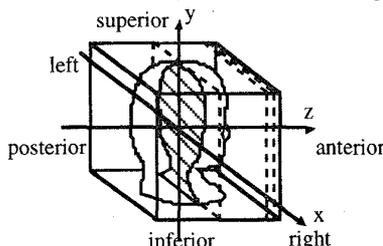


Figure 5 Coordinate system.

(i) **Location knowledge:** For each axis of x, y and z, Table 2 tabulates which location every ROI belongs to. In it, N, Ze and P express Negative, Zero and Positive membership functions, respectively, in Figure 6. The horizontal axis corresponds to arrows, x, y, or z, in Figure 5. The membership functions VH, H, L and VL are shown in Figure 6.

Table 2 A knowledge table on the ROIs.

	ROI	μ_{LCH}	μ_{RCH}	μ_{CB}	μ_{BS}
X	N	VH	VL	VH	VL
	Ze	L	H	VH	VH
	P	VL	VH	VH	VL
Y	N	L	L	VH	VH
	Ze	H	H	H	H
	P	VH	VH	VH	VL
Z	N	VH	VH	VH	VL
	Ze	VH	VH	H	VH
	P	VH	VH	VH	L

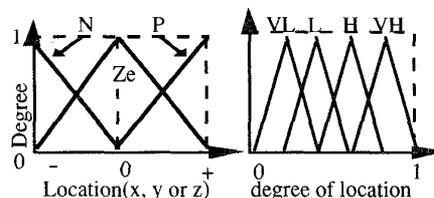


Figure 6 The membership functions.

The notation μ_X denotes the membership function of the ROI X. From Table 2, the rules of x-axis of LCH is derived:

If $x=N$ (left) then $\mu_{LCH} = VH$,

else if $x=Ze$ (middle) then $\mu_{LCH} = L$,

else if $x=P$ (right) then $\mu_{LCH} = VL$.

For a given voxel with coordinate (a, b, c), a fuzzy inference result, U_{LCH} , is obtained via a min-max inference,

$$U_{LCH}(a) = [a \wedge N \wedge VH] \vee [a \wedge Ze \wedge L] \vee [a \wedge P \wedge VL].$$

The center of gravity, $\mu_{LCH}(a)$, of U_{LCH} is obtained. We can also obtain $\mu_{RCH}(a)$, $\mu_{CB}(a)$ and $\mu_{BS}(a)$ for a of (a, b, c). Similarly, we can obtain the values, $\mu_{LCH}(b)$ and $\mu_{LCH}(c)$, for b and c of (a, b, c), respectively. Finally, we determine a degree $\mu_{LCH}(a,b,c) = \mu_{LCH}(a) \wedge \mu_{LCH}(b) \wedge \mu_{LCH}(c)$, for the voxel as the degree for the left cerebral hemisphere. Similarly, the degrees $\mu_{RCH}(a,b,c)$, $\mu_{CB}(a,b,c)$ and $\mu_{BS}(a,b,c)$, for the other ROIs are also obtained. Thus we can determine a set $[\mu_{LCH}(a,b,c), \mu_{RCH}(a,b,c), \mu_{CB}(a,b,c), \mu_{BS}(a,b,c)]$ for every voxel.

For the obtained set,

IF only one degree, μ_p , of the set is larger than 0.5, we determine the voxel belongs to the ROI "p".

ELSE, we employ the fuzzy inferences of (ii) and (iii)

(ii) Boundary knowledge

a) Proximity: The difference between μ_p and μ_q can serve an indicator of the boundary surface. The degree

$$\mu_1^{Pr}(P, Q) \text{ is defined as } (1 - |\mu_p - \mu_q|) \times \mu_p \times \mu_q.$$

b) Euclidean distance: There exists the narrow part between adjacent ROIs. We calculate the Euclidean distance, \mathcal{E} , from the voxel to the outside of the brain region (background region). The voxel closer to the boundary surface implies the higher possibility of boundary. The notation μ_B^E denotes the degree of the boundary, then,

If the distance \mathcal{E} is L then μ_B^E is HI,

else if the distance \mathcal{E} is M then μ_B^E is MI,

else if the distance \mathcal{E} is H then μ_B^E is LO.

The center of gravity, μ_2^E , of the obtained inference result is a degree of the boundary surface. The membership functions L, M, H and HI, MI, LO are shown in Figure 7.

(iii) **Intensity knowledge** : The boundary surface mainly consists of the gray matter. The notation μ_B^I denotes a boundary information on intensity on the gray matter. Then,

If the intensity is L then μ_B^I is HI,

else if the intensity is M then μ_B^I is MI,

else if the intensity is H then μ_B^I is LO.

The center of gravity, μ_3^I , of the inference result is a degree of boundary surface with respect to intensity.

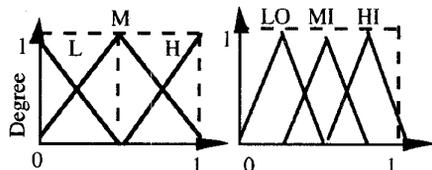


Figure 7 L, M, H and HI, MI and LO membership functions.

Table 2 The weight table.

Pair <i>i</i>	ROI P	ROI Q	ω_i^α	ω_i^β	ω_i^γ
1	LCH	RCH	1	1	0
2	LCH	CB	0	1	1
3	LCH	BS	1	1	0
4	RCH	CB	0	1	1
5	RCH	BS	1	1	0
6	CB	BS	1	1	0

Combine of three inference results by

$$\mu_t = \max_{\{pair\ i : i=1..6\}} \{\omega_i^\alpha \mu_i^{Pr}(P,Q) + \omega_i^\beta \mu_2^E + \omega_i^\gamma \mu_3^I\}$$

Since $\mu_i^P(P,Q) = \mu_i^Q(Q,P)$, we can consider all cases with Table 2. Thus obtaining a total degree from all inference results. Finally, we construct the voxels on the region growing by including smaller μ_t as the ROI.

Results: We applied the method to 36 MR voxel data. Our method can segment all voxel data with average error ratio 2.7 %. It required about three minutes on each MR voxel data on IRIS Indigo2 (IRIX 5.3, 150Mz, 128M).

3.2.2 Human Foot CT images

This subsection is to segment the whole bone region into their ossa. The CT images are obtained from Somatom Plus-S CT scanner (Siemens Erlangen, Germany). Field of view was 220mm. Matrix was 512 by 512. This generated 64, 3.44mm-thick, contiguous section images; giving voxel dimensions of $0.43 \times 0.43 \times 3.44 \text{ mm}^3$. We construct the voxel data consisted of $64 \times 512 \times 512$ voxels. The intensity ranged between 0 and 2048. First, we extract the whole bone region based on the region growing algorithm with threshold 1180.

(i) **Boundary knowledge:** The voxel can be closer to the narrow part as the Euclidean distance becomes smaller. We can then derive the following rules in the similar way in

the case of Sect. 3.2.1. The notation μ^B denotes a degree of boundary surface for the segmentation. Then,

If the distance \mathcal{E} is L then μ^B is HI, else if \mathcal{E} is M then μ^B is MI, else \mathcal{E} is H then μ^B is LO.

The center of gravity, μ_1^E , of the obtained inference result is a degree of the boundary surface.

(ii) **Intensity knowledge** : The intensity of bone edges has high. Therefore,

If the intensity is L then μ^B is LO, else if the intensity is M then μ^B is MI, else the intensity is H then μ^B is HI.

The center of gravity, μ_2^I , of the obtained inference result is a degree of boundary surface with respect to intensity.

(iii) **Joint knowledge** : Figure 8 shows the joint part intensity distribution. A value of $J=(lmax-min)(rmax-min)$ can indicate the joint knowledge. The notation μ^J denotes a degree of joint knowledge of bones. Then,

If the J is L then μ^J is LO, else if J is M then μ^J is MI, else J is H then μ^J is HI.

The center of gravity, μ_3^J , of the obtained inference result is a degree of Joint knowledge.

Combine of three inference results by

$$\mu_t = \frac{\mu_1^E + \mu_2^I + \mu_3^J}{3}$$

Thus we obtain a total degree from all inference results. Finally, we construct the voxels on the region growing by including smaller μ_t as the ROI.

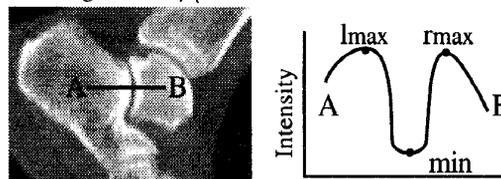


Figure 8 Joint part image and the intensity distribution.

Results: We can correctly segment five images of seven CT data sets. A comparison with genuine ROIs from the viewpoint of anatomy shows that our method precisely segmented them. It required about ten minutes on each 3D human foot CT voxel data on the IRIS Indigo2.

4. FUZZY CLUSTER IDENTIFICATION

4.1. Preliminaries

We obtained the MR angiography images from the MRI scanner. The image acquisition method was 3D TOF Angiography with TR = 61 msec and TE = 3 msec. Field of View (FOV) was 120 mm. Matrix was 256 by 256. Thickness of the slice was 0.8 mm. The dimension of given voxel was $0.47 \times 0.47 \times 0.8 \text{ mm}^3$. In our work, the feature images appear and treated as intensity images. The intensity of all intracranial structure ranged between 0 and 4096. We construct the clusters with continuous hierarchy

clustering. We define a similarity measure between two clusters with the within-class variance and the between-class variance. Our hierarchy clustering is : Consider the number of clusters be equal to the number of voxels; For all combination of two clusters connecting with each other, we calculate the similarity measure; Combine two clusters with minimum similarity measure into one cluster. In the result we obtain hundreds of clusters.

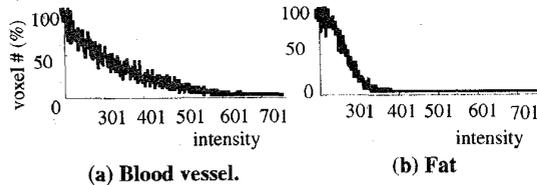


Figure 9 Typical intensity histogram.

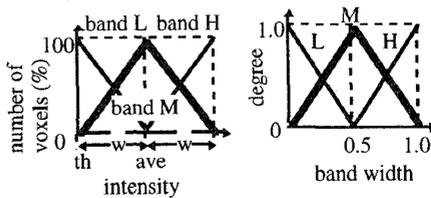


Figure 10 Three bands and the membership functions.

4. 2 Cluster Identification

The clusters obtained in Section 4.1 has coherent intensity distributions, that is, the blood vessel region consists of many voxels with high intensity and fat region consists of many voxels with low intensity. These intensity distributions overlap each other. To recognize intensity distribution of the cluster, we can make a model expressing the coherent intensity distributions in fuzzy languages. Typical intensity of the blood vessel and the fat are shown in Figure 9. Whole range of intensity distribution is divided into three bands, "band L," "band M" and "band H", and each of them can be represented by fuzzy languages, "Low, L" "Middle, M" and "High, H", as shown in Figure 10. In the case of the blood vessel, the fuzzy model of intensity distribution is expressed by $Model_{blood}(band\ L, band\ M, band\ H) = (don't\ care, middle, high)$. In the case of the fat, $Model_{fat} = (high, middle, low)$. For an interested cluster $X=(X_1, X_2, X_3)$, we make up the intensity distribution and we calculate the minimum among three minimums for all bands. We classify the clusters into either the blood vessels or other tissues by picking up larger one of the minimums. Thus, we can identify all clusters to be blood vessel or fat.

Results: We apply our method to some 3D TOF MRA data. Our cluster identification successfully recognizes every cluster. The results show that our method can successfully segment the blood vessel region.

5. CONCLUSIONS

This paper introduced fuzzy logic approaches to segment human brain MR, foot CT and MRA images. Figure 11

summarized these segmentation results. Thus, fuzzy logic can provide powerful tools to manipulate human knowledge.

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